

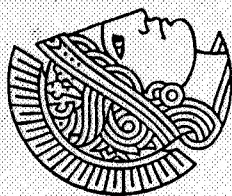
Schriften zur Quantitativen Betriebswirtschaftslehre
Technische Hochschule
Darmstadt

Fachgebiete

Operations Research • Prof. Dr. W. Domschke

Fertigungs- und Materialwirtschaft • Prof. Dr. H. Stadler

Betriebliche Kommunikationssysteme • Prof. Dr. H.J. Petzold



Armin Scholl

Data of Assembly Line Balancing Problems

Nummer 16/93

November 1993

Letzte Änderung: September 1997

Dr. Armin Scholl

Technische Universität Darmstadt
Institut für Betriebswirtschaftslehre
Hochschulstraße 1, D - 64289 Darmstadt
Tel. 06151/164564, Fax: 06151/166504
E-Mail: scholl@bwl.th-darmstadt.de

Data of Assembly Line Balancing Problems

Armin Scholl

Technische Hochschule Darmstadt, Institut für Betriebswirtschaftslehre
Hochschulstraße 1, D - 64289 Darmstadt, Germany

In this paper, we present benchmark data sets for Simple Assembly Line Balancing Problems (SALBP) which may be used to test and compare solution procedures. The (general) SALBP consists of assigning tasks to work stations of an assembly line such that the sum of idle times is minimized. The number m of stations as well as the production rate, or equivalently, the cycle time c have to be determined simultaneously. Additionally, precedence constraints between the tasks have to be considered. Two restricted versions of the problem, often considered in the literature, are SALBP-1 and SALBP-2. SALBP-1 is to minimize the number of stations for a given cycle time whereas SALBP-2 is to minimize the cycle time for a given number of stations.

1. Introduction to SALBP

The simple assembly line balancing problem (SALBP) arises in the mass production of a single product on a paced assembly line. The production process is subdivided into n tasks (indivisible elements of work) whose operation times t_j ($j = 1, \dots, n$) are assumed to be integral constants. Due to technological restrictions, precedence constraints partially specifying the sequence of tasks have to be considered. These constraints can be represented by a precedence graph containing nodes for all tasks with operations times as node weights and arcs (i, j) whenever a task i has to be completed before a task j can be started. The assembly line consists of a sequence of m work stations which are connected by a conveyor belt. Each station repeatedly performs a subset of the tasks on consecutive product units moving along the line at constant speed. The fixed time interval available for operating a unit in each station is called cycle time c . The production rate of the assembly line is given by $1/c$.

The general type of the simple assembly line balancing problem (SALBP-G for short) can be stated as follows (see, e.g., Baybars (1986) or Domschke et al. (1993, ch. 4)). Each task has to be assigned to one station of the assembly line in such a way that no precedence constraint is violated. Equivalently, a partition of the set of all tasks into disjoint sets S_k (station loads) with $k = 1, \dots, m$ is to be found. For each arc (i, j) of the precedence graph the relation $h \leq k$ must hold if $i \in S_h$ and $j \in S_k$. The station time $t(S_k)$ is equal to the sum of operation times of all tasks which are included in the set S_k . The maximum station time determines the (realized) cycle time c resulting in an idle time $c - t(S_k)$ for all stations k with a station time lower than c . The objective is to find a cycle time c and a number m of stations as well as a corresponding task assignment such that the total idle time (balance delay) is minimized. Usually, upper and lower bounds on c and m have to be considered.

The objective function can be written as "Minimize $m \cdot c - t_{\text{sum}}$ ", where t_{sum} denotes the sum of operation times. Since this function is nonlinear depending on the variables m and c , the following versions of SALBP are often considered:

- SALBP-1: For a given cycle time c , the number m of stations has to be minimized.
- SALBP-2: For a given number m of stations, the cycle time c has to be minimized.

Since the production rate is a parameter of SALBP-1, this problem type is given whenever a new assembly line system has to be installed and the external demand can be well estimated. In contrast, SALBP-2 leads to the maximization of the production rate of an existing assembly line when changes in the production process take place. These changes may concern reductions of operation times or other variations of the assembly plan.

A further restricted version of SALBP may be denoted by SALBP-F (feasibility problem): The number m of stations and the cycle time c are given. The feasibility problem is to decide whether a feasible assignment of all tasks to the m stations with the maximum station time not exceeding the cycle time c exists.

The remainder of the paper is organized as follows. Section 2 discusses characteristics and complexity measures of SALBP. In Section 3, a collection of precedence graphs, which are completely defined in the appendix, is described. In Section 4, we present benchmark data sets for SALBP-1 used in the literature. Furthermore, a new challenging data set is introduced. New benchmark data sets for SALBP-2, which are based on the same precedence graphs as the data sets for SALBP-1, are defined in Section 5. Section 6 contains the description of a new benchmark data set for SALBP-G.

2. Characteristics of SALBP and Complexity Measures

SALBP-G as well as its restricted versions SALBP-1, SALBP-2, and SALBP-F, respectively, are NP-hard, because they can be reduced to the problem *partition* (cf. Karp (1972)). Nevertheless, reasonable solution procedures usually solve a large number of corresponding instances in short computation times, while others are computationally intractable in terms of available computation time. Therefore, one is interested in measures which indicate whether or not a certain problem instance will probably take much computation time. Such information may help deciding on the solution procedure to be used, especially if exact or heuristic methods are more likely to obtain reasonable solutions within acceptable computation times. In the sequel, we describe characteristics of SALBP and argue their expected influences on the complexity of problem instances. In order to quantify those complexity aspects, we present some numerical measures of problem characteristics and discuss their suitability as complexity indicators.

2.1 Problem characteristics

Following properties of problem data and optimal solutions may have influence on the complexity of a problem instance.

Number n of tasks: SALBP falls into the general class of sequencing problems (cf. Baker (1974)). Ignoring precedence constraints, there are $n!$ feasible task sequences. Therefore, the complexity is expected to grow exponentially with increasing n if the precedence constraints are not too restrictive (see below).

Operation times and cycle time: If operation times are small compared with the cycle time, there should be more station loads with low idle times and probably more optimal solutions than in the case of large operation times, because combining small items is usually easier than combining large ones. Furthermore, optimal solutions tend to have objective function values equal to a lower bound. This may be advantageous for optimum seeking procedures. Additionally, it can be expected that operations with significantly varying operation times (including rather small values) can be combined to form station loads with low idle times better than such with almost identical operation times. This is especially true if these times are rather large relative to the cycle time. However, an instance with identical operation times for all tasks is simple.

Precedence constraints: These constraints have an influence which is twofold. On the one hand, they restrict the number of feasible sequences which may lead to the assumption that the complexity decreases with increasing number of precedence relations. On the other hand, such constraints may complicate the process of finding optimal solutions, because the loads of some stations $1, \dots, k$ determine the tasks which are assignable to the remaining stations $k+1, k+2, \dots, m$. Therefore, the decision on a current station load is more sensitive than it is in the absence of precedence constraints. As a consequence, not only the quantity but also the structure of precedence relations has an influence on the complexity. For enumeration procedures which load the stations in the order $k = 1, 2, \dots$, it should be advantageous if only few alternative station loads exist for the early stations, in particular with respect to the sensitivity of assignment decisions. This requires that the precedence graph has only few sources and some type of diverging structure, i.e., the number of direct successors tends to increase when approaching the sinks of the graph. For such structures, there exist only few tasks potentially assignable to the first stations, while more tasks get assignable to later stations. Since the sensitivity of assignment decisions decreases from early to late stations due to the decreasing number of available (unassigned) tasks, it can be expected that problem instances with diverging precedence structures are less complex than those with other structures.

Number m of stations: The influence of the number m of stations on the complexity is not unique. The less stations are available, the more combinations of tasks have to be examined for each station (with fixed n). As a consequence, one may expect that the complexity

increases if m decreases. However, the number of stations available for the assignment of each task decreases, too, especially if there are precedence constraints. Additionally, the probability that the objective function value of the optimal solution is equal to a lower bound is large if many different tasks are potentially assignable to a station. Note that the extreme cases $m = 1$ and $m = n$ are trivial.

2.2 Measures of Complexity

In the following, we present simple numerical measures (besides the number n of tasks) of the above mentioned problem characteristics and discuss their suitability as indicators for the complexity of problem instances (cf., additionally, Baybars (1986)).

Order strength (cf. Mastor (1970), Bhattacharjee and Sahu (1990)): The order strength $OS \in [0,1]$ is defined as the number of arcs in the transitive closure of the precedence graph divided by $n \cdot (n-1)/2$, the maximal number of arcs in an acyclic graph with n nodes. That is, OS measures the relative number of precedence relations. Problems with a large order strength are basically expected to be more complex than such with small OS values. However, this assumption is critical as mentioned in the above discussion on the influence of precedence constraints. Note that only one task sequence is feasible if $OS = 1$. In the case of $OS = 0$, SALBP-1 reduces to the bin packing problem and SALBP-2 to the parallel machine scheduling problem with the objective of minimizing the makespan, which are both NP-hard.

Flexibility ratio (cf. Dar-El (1973)): The flexibility ratio $FR \in [0,1]$ is defined as the number of zero entries in the (transitive) precedence matrix divided by the total number of entries. This measure is equivalent to the order strength because of $FR = 1 - OS$.

West ratio (cf. Dar-El (1973)): The west ratio $WR = n/m$ is the average number of tasks per station. In SALBP-1 and SALBP-G, this value depends on the optimum number of stations or even a lower bound. The above discussion on the influence of m on the complexity implies that problem instances with small values of WR tend to be more complex than such with larger values.

Time interval (cf. Wee and Magazine (1981)): The interval $TI = [t_{\min}/c, t_{\max}/c] \subseteq [0,1]$ is a measure of the interrelation between the cycle time and the operation times. For SALBP-2 and SALBP-G, this measure depends on the minimal cycle time or a lower bound. A small length of TI indicates that the operation times vary only in a small range. Furthermore, the position of TI in the interval $[0,1]$ shows if the operation times are large or small relative to the cycle time c . Following the above argumentation, problems are expected to be relatively complex if TI is small and has a position near to the right border of $[0,1]$.

Time variability ratio: A measure similar to the time interval is defined by $TV = t_{\max}/t_{\min}$. In contrast to TI , only one value is used to characterize a problem instance. Furthermore, this measure is independent of the cycle time and, therefore, reflects

the time structure of a precedence graph rather than single problem instances. Small values of TV indicate that the operation times vary only in a small range or that the minimal operation time is remarkably large. Following the above discussion on the time interval, the complexity of problem instances is expected to grow with decreasing values of TV.

The above measures have the major drawback that each of them controls only a part of the problem characteristics which may influence the complexity. Therefore, in the literature, it is proposed to choose a combination of measures (e.g. number of tasks, order strength, west ratio; cf., e.g., Johnson (1988)). Nevertheless, the interrelations between several characteristics which may enforce or balance their influences on the complexity cannot be considered. Furthermore, the order strength is only a very rough measure, because it does not reflect the structure of the precedence graph. Hence, there is a need for further measures which utilize the information provided by the precedence structure more intensively.

3. Collection of Precedence Graphs

Name	Reference	n	t _{min}	t _{max}	t _{sum}	OS	TV
Arcus 1	Arcus (1963)	83	233	3691	75707	59.1	15.8
Arcus 2	Arcus (1963)	111	10	5689	150399	40.4	568.9
Bartholdi 1	Bartholdi (1993)	148	3	383	5634	25.8	127.7
Bartholdi 2	Bartholdi (1993), modified	148	1	83	4234	25.8	83
Bowman	Bowman (1960)	8	3	17	75	75	5.7
Buxey	Buxey (1974)	29	1	25	324	50.7	25
Gunther	Gunther et al. (1983)	35	1	40	483	59.5	40
Hahn	Hahn (1972)	53	40	1775	14026	83.8	44.4
Heskiaoff	Heskiaoff (1968)	28	1	108	1024	22.5	108
Jackson	Jackson (1956)	11	1	7	46	58.2	7
Jaeschke	Jaeschke (1964)	9	1	6	37	83.3	6
Kilbrid	Kilbridge and Wester (1962)	45	3	55	552	44.6	18.3
Lutz 1	Lutz (1974)	32	100	1400	14140	83.5	14
Lutz 2	Lutz (1974), modified	89	1	10	485	77.6	10
Lutz 3	Lutz (1974)	89	1	74	1644	77.6	74
Mansoor	Mansoor (1964)	11	2	45	185	60	22.5
Mertens	Mertens (1967)	7	1	6	29	52.4	6
Mitchell	Mitchell (1957)	21	1	13	105	71	13
Mukherjee	Mukherjee and Basu (1964)	94	8	171	4208	44.8	21.4
Rosenberg	Rosenberg and Ziegler (1992)	25	1	13	125	71.7	13
Sawyer	Sawyer (1970)	30	1	25	324	44.8	25
Scholl	real world problem	297	5	1386	69655	58.2	277.2
Tonge	Tonge (1961)	70	1	156	3510	59.4	156
Warnecke	Warnecke (1971), modified	58	7	53	1548	59.1	7.6
Wee - Mag	Wee and Magazine (1981)	75	2	27	1499	22.7	13.5

TABLE 1. Characterization of Precedence Graphs.

In this section, we specify the precedence graphs used to define data sets for the different versions of SALBP. These graphs which are partly based on real world problems have been collected from the literature (except one). Some of them have been modified or complemented. In Table 1, the original reference, the number n of tasks, the minimal and the

maximal operation time as well as the sum of operation times are given for each graph. Furthermore, the values of the two complexity measures order strength OS (in %) and time variability ratio TV are provided. The precedence relations and operation times are completely specified in the appendix. Information on cycle times and numbers of stations are added in Sections 4 to 6 resulting in different problem instances for each precedence graph.

4. Benchmark Data Sets for SALBP-1

We give benchmark data sets which are used in the literature to evaluate and compare algorithms and provide optimal (or best known) objective function values. Furthermore, we develop a new data set involving precedence graphs which are more challenging than those in former data sets.

4.1 Data Set of Talbot et al.

This data set contains 12 precedence graphs with 8 to 111 tasks. By adding various cycle times, Talbot et al. (1986) generated 64 instances. In order to give a compact presentation, the instances of each graph are grouped into five columns in Table 2. For each instance, we give the cycle time c and the minimal number of stations m^* . If m^* is not equal to the theoretical minimum number of stations $m_{\min} := \lceil t_{\text{sum}}/c \rceil$, the symbol '+' is added.

Name	c	m*	c	m*	c	m*	c	m*	c	m*
Bowman	20	5 ⁺								
Mansoor	48	4	62	3	94	2				
Mertens	6	6 ⁺	7	5	8	5 ⁺	10	3	15	2
	18	2								
Jaeschke	6	8 ⁺	7	7 ⁺	8	6 ⁺	10	4	18	3
Jackson	7	8 ⁺	9	6	10	5	13	4	14	4
	21	3								
Mitchell	14	8	15	8 ⁺	21	5	26	5	35	3
	39	3								
Heskiaoff	138	8	205	5	216	5	256	4	324	4
	342	3								
Sawyer	25	14 ⁺	27	13 ⁺	30	12 ⁺	36	10 ⁺	41	8
	54	7 ⁺	75	5						
Kilbrid	57	10	79	7	92	6	110	6	138	4
	184	3								
Tonge	176	21 ⁺	364	10	410	9	468	8	527	7
Arcus 1	5048	16 ⁺	5853	14 ⁺	6842	12	7571	11 ⁺	8412	10 ⁺
	8898	9	10816	8 ⁺						
Arcus 2	5755	27	8847	18 ⁺	10027	16 ⁺	10743	15 ⁺	11378	14
	17067	9								

TABLE 2. Data Set of Talbot et al.

4.2 Data Set of Hoffmann

The data set of Talbot et al. does not provide a hard test, because most instances are easily solved by common algorithms. Therefore, Hoffmann (1990, 1992) tried to find more strenuous instances by considering cycle times which would cause a total idle time close to zero if the theoretical minimum number of stations m_{\min} is realized. These instances are expected to have an optimal solution with more than m_{\min} stations or only few optimal solutions with that number of stations. Hoffmann used the following procedure to determine the cycle times:

$$m_{\max} := \lfloor t_{\text{sum}}/t_{\max} \rfloor;$$

$$c(m) := \lceil t_{\text{sum}}/m \rceil \quad \text{for } m = \lfloor m_{\max}/2 \rfloor, \dots, m_{\max}$$

For the precedence graphs *Arcus 1*, *Arcus 2*, *Kilbrid*, *Sawyer*, and *Tonge*, a data set with 50 instances results (see Table 3). If m^* is currently not available for an instance, the best known values of a lower and an upper bound on m^* are given (separated by a hyphen).

Name	c	m^*	c	m^*	c	m^*	c	m^*	c	m^*
Sawyer	27	13 ⁺	30	12 ⁺	33	11 ⁺	36	10 ⁺	41	8
	47	7	54	7 ⁺						
Kilbrid	56	10	62	9	69	8	79	7	92	6
	111	5								
Tonge	160	23 ⁺	168	22 ⁺	176	21 ⁺	185	20 ⁺	195	19 ⁺
	207	18 ⁺	220	17 ⁺	234	16 ⁺	251	14	270	14 ⁺
	293	13 ⁺	320	11						
Arcus 1	3786	21 ⁺	3985	20 ⁺	4206	19 ⁺	4454	18 ⁺	4732	17 ⁺
	5048	16 ⁺	5408	15 ⁺	5824	14 ⁺	6309	13 ⁺	6883	12 ⁺
	7571	11 ⁺								
Arcus 2	5785	27 ⁺	6016	26 ⁺	6267	25 ⁺	6540	24 ⁺	6837	23 ⁺
	7162	22 ⁺	7520	20-21	7916	20 ⁺	8356	19 ⁺	8847	18 ⁺
	9400	17 ⁺	10027	16 ⁺	10743	15 ⁺	11570	13		

TABLE 3. Data Set of Hoffmann.

Additionally, Hoffmann checked the precedence graphs *Tonge* for all cycle times between 156 and 1746 (23 to 3 stations) and *Sawyer* for all cycle times between 25 and 161 (14 to 3 stations). Only few instances of both graphs (in particular those which are contained in the above table) are hard instances in terms of computation time requirements.

4.3 New Data Set for SALBP-1

We propose a data set with 168 instances which includes precedence graphs not yet considered in other data sets. Cycle times are determined by the method of Hoffmann (see Section 4.2). In contrast to the data sets in Section 4.1 and 4.2, there are problem instances with the difference $m^* - m_{\min}$ being larger than 1. In those cases, this difference is specified following the '+' sign.

Name	c	m*	c	m*	c	m*	c	m*	c	m*
Rosenberg	14	10 ⁺	16	8	18	8 ⁺	21	6	25	6 ⁺
	32	4								
Buxey	27	13 ⁺	30	12 ⁺	33	11 ⁺	36	10 ⁺	41	8
	47	7	54	7 ⁺						
Lutz 1	1414	11 ⁺	1572	10 ⁺	1768	9 ⁺	2020	8 ⁺	2357	7 ⁺
	2828	6 ⁺								
Gunther	41	14 ⁺²	44	12 ⁺	49	11 ⁺	54	9	61	9 ⁺
	69	8 ⁺	81	7 ⁺						
Hahn	2004	8	2338	7	2806	6	3507	5	4676	4
Warnecke	54	31 ⁺²	56	29 ⁺	58	29 ⁺²	60	27 ⁺	62	27 ⁺²
	65	25 ⁺	68	24 ⁺	71	23 ⁺	74	22 ⁺	78	21 ⁺
	82	20 ⁺	86	19 ⁺	92	17	97	17 ⁺	104	15
	111	14								
Wee - Mag	28	63 ⁺⁹	29	63 ⁺¹¹	30	62 ⁺¹²	31	62 ⁺¹³	32	61 ⁺¹⁴
	33	61 ⁺¹⁵	34	61 ⁺¹⁶	35	60 ⁺¹⁷	36	60 ⁺¹⁸	37	60 ⁺¹⁹
	38	60 ⁺²⁰	39	60 ⁺²¹	40	60 ⁺²²	41	59 ⁺²²	42	55 ⁺¹⁹
	43	50 ⁺¹⁵	45	38 ⁺²	46	34 ⁺	47	32-33	49	32 ⁺
	50	32 ⁺²	52	31 ⁺²	54	31 ⁺³	56	30 ⁺³		
Lutz 2	11	49 ⁺⁴	12	44 ⁺³	13	40 ⁺²	14	37 ⁺²	15	34 ⁺
	16	31	17	29	18	28 ⁺	19	26	20	25
	21	24								
Lutz 3	75	23 ⁺	79	22 ⁺	83	21 ⁺	87	20 ⁺	92	19 ⁺
	97	18 ⁺	103	17 ⁺	110	15	118	14	127	14 ⁺
	137	13 ⁺	150	12 ⁺						
Mukherjee	176	25 ⁺	183	24 ⁺	192	23 ⁺	201	22 ⁺	211	21 ⁺
	222	20 ⁺	234	19 ⁺	248	18 ⁺	263	17 ⁺	281	16 ⁺
	301	15 ⁺	324	14 ⁺	351	13 ⁺				
Bartholdi 1	403	14	434	13	470	12	513	11	564	10
	626	9	705	8	805	7				
Bartholdi 2	84	51	85	50-51	87	49	89	48	91	47
	93	46	95	45	97	44	99	43	101	42
	104	41	106	40	109	39	112	38	115	37
	118	36	121	35	125	34	129	33	133	32
	137	31	142	30	146	29	152	28	157	27
	163	26	170	25						
Scholl	1394	50-51	1422	50 ⁺	1452	48	1483	47-48	1515	46-47
	1548	46 ⁺	1584	44	1620	44 ⁺	1659	42-43	1699	41-42
	1742	40	1787	39-40	1834	38	1883	37	1935	36
	1991	35	2049	34	2111	33	2177	32	2247	31
	2322	30	2402	29	2488	28	2580	27	2680	26
	2787	25								

TABLE 4. Data Set of Scholl.

5. Benchmark Data Sets for SALBP-2

In the literature, there is no benchmark data set available for SALBP-2. We present two new data sets consisting of the precedence graphs of Table 1. For each graph, a range $[\underline{m}, \bar{m}]$ of

station numbers is defined resulting in up to $\bar{m}-\underline{m}+1$ different instances. Generally, the ranges are restricted by $\underline{m} \geq 3$ and $\bar{m} \leq \min \{ \lceil t_{\text{sum}}/t_{\text{max}} \rceil + 2, \lfloor n/2 \rfloor \}$. Additionally, some easy instances are eliminated and the number of instances per precedence graph is limited to 28 in order to prevent one problem structure dominating the whole data set. Therefore, in some cases only a subrange of $[\underline{m}, \bar{m}]$ is used.

5.1 Data Set 1 for Type 2

The first data set for SALBP-2 includes all graphs with $n \geq 30$ of the data sets for SALBP-1 in Sections 4.1 and 4.2. Additionally, it contains the graphs *Buxey*, *Gunther*, *Lutz 1*, and *Lutz 2* resulting in a total of 128 problem instances.

For each graph, the different instances are grouped into five columns. For each value of m , the minimum cycle time c^* is specified. If c^* is not yet known, an interval of possible values is provided (for the upper bounds only the last digits which differ from the lower bound values are noted). Instances for which c^* is larger than the theoretical minimum cycle time $c_{\text{min}} := \max \{ \lceil t_{\text{sum}}/m \rceil, t_{\text{max}} \}$ are marked with '+'. If the difference $c^* - c_{\text{min}}$ is larger than 1, its value is added after the '+' sign.

Name	m	c*	m	c*	m	c*	m	c*	m	c*
Buxey	7	47	8	41	9	37 ⁺	10	34 ⁺	11	32 ⁺²
	12	28 ⁺	13	27 ⁺²	14	25				
Gunther	6	84 ⁺³	7	72 ⁺	8	63 ⁺²	9	54	10	50 ⁺
	11	48 ⁺⁴	12	44 ⁺³	13	42 ⁺²	14	40	15	40
Kilbrid	3	184	4	138	5	111	6	92	7	79
	8	69	9	62	10	56	11	55		
Lutz 1	8	1860 ⁺⁹²	9	1638 ⁺⁶⁶	10	1526 ⁺¹¹²	11	1400	12	1400
Lutz 2	9	54	10	49	11	45	12	41	13	38
	14	35	15	33	16	31	17	29	18	28 ⁺
	19	26	20	25	21	24	22	23	23	22
	24	21	25	20	26	19	27	19 ⁺	28	18
Sawyer	7	47	8	41	9	37 ⁺	10	34 ⁺	11	31 ⁺
	12	28 ⁺	13	26 ⁺	14	25				
Tonge	3	1170	4	878	5	702	6	585	7	502
	8	439	9	391 ⁺	10	352 ⁺	11	320	12	294 ⁺
	13	271 ⁺	14	251	15	235 ⁺	16	221 ⁺	17	208 ⁺
	18	196 ⁺	19	186 ⁺	20	177 ⁺	21	170 ⁺²	22	162 ⁺²
	23	156	24	156	25	156				
Arcus 1	3	25236	4	18927	5	15142	6	12620 ⁺²	7	10826 ⁺¹⁰
	8	9554 ⁺⁹⁰	9	8499 ⁺⁸⁷	10	7580 ⁺⁹	11	7084 ⁺²⁰¹	12	6412 ⁺¹⁰³
	13	5864 ⁺⁴⁰	14	5441 ⁺³³	15	5104 ⁺⁵⁶	16	4850 ⁺¹¹⁸	17	4516 ⁺⁶²
	18	4317 ⁺¹¹¹	19	4068 ⁺⁸³	20	3882 ⁺⁹⁶	21	3691	22	3691
Arcus 2	3	50133	4	37600	5	30080	6	25067	7	21486
	8	18800	9	16711	10	15040-41	11	13673	12	12534
	13	11570	14	10747 ⁺⁴	15	10035-36 ⁺⁸	16	9411-14 ⁺¹¹	17	8854-61 ⁺⁷
	18	8376-77 ⁺²⁰	19	7920-28 ⁺⁴²	20	7520-26	21	7181-203 ⁺¹⁹	22	6839-59 ⁺²
	23	6551-79 ⁺¹¹	24	6273-89 ⁺⁶	25	6085-106 ⁺⁶⁹	26	5844-56 ⁺⁵⁹	27	5689

TABLE 5. First Data Set for SALBP-2.

5.2 Data Set 2 for Type 2

The second data set contains graphs with $n \geq 53$ tasks and a total of 174 instances.

Name	m	c*	m	c*	m	c*	m	c*	m	c*
Hahn	3	4787 ⁺¹¹¹	4	3677 ⁺¹⁷⁰	5	2823 ⁺¹⁷	6	2400 ⁺⁶²	7	2336 ⁺³³²
	8	1907 ⁺¹³²	9	1827 ⁺⁵²	10	1775				
War- necke	3	516	4	387	5	310	6	258	7	222
	8	194	9	172	10	155	11	142 ⁺	12	130 ⁺
	13	120	14	111	15	104	16	98 ⁺	17	92
	18	87 ⁺	19	84 ⁺²	20	79 ⁺	21	76 ⁺²	22	72-73 ⁺
	23	69 ⁺	24	66 ⁺	25	64 ⁺²	26	64 ⁺²	27	60 ⁺²
	28	59 ⁺³	29	56 ⁺²						
Wee- Mag	3	500	4	375	5	300	6	250	7	215
	8	188	9	167	10	150	11	137	12	125
	13	116	14	108	15	100	16	94	17	89
	18	84-87	19	84-85 ⁺⁵	20	77 ⁺²	21	72	22	69
	23	66-67	24	66 ⁺³	25	66 ⁺⁶	26	65 ⁺⁷	27	63-65 ⁺⁷
	28	63-64 ⁺⁹	29	63 ⁺¹¹	30	56 ⁺⁶				
Lutz 3	3	548	4	411	5	329	6	275 ⁺	7	236 ⁺
	8	207 ⁺	9	184 ⁺	10	165	11	151 ⁺	12	138 ⁺
	13	128 ⁺	14	118	15	110	16	105 ⁺²	17	98 ⁺
	18	93 ⁺	19	89 ⁺²	20	85 ⁺²	21	80 ⁺	22	76 ⁺
	23	74								
Mukher- jee	3	1403	4	1052	5	844 ⁺²	6	704 ⁺²	7	621 ⁺¹⁹
	8	532 ⁺⁶	9	471 ⁺³	10	424 ⁺³	11	391 ⁺⁸	12	358 ⁺⁷
	13	325 ⁺	14	311 ⁺¹⁰	15	288 ⁺⁷	16	268 ⁺⁵	17	251 ⁺³
	18	239 ⁺⁵	19	226 ⁺⁴	20	220 ⁺⁹	21	208 ⁺⁷	22	200 ⁺⁸
	23	189 ⁺⁶	24	179 ⁺³	25	172 ⁺¹	26	171		
Bart- holdi 1	3	1878	4	1409	5	1127	6	939	7	805
	8	705	9	626	10	564	11	513	12	470
	13	434	14	403	15	383				
Bart- holdi 2	27	157	28	152	29	146	30	142	31	137
	32	133	33	129	34	125	35	121	36	118
	37	115	38	112	39	109	40	106	41	104
	42	101	43	99	44	97	45	95	46	93
	47	91	48	89	49	87	50	85-86	51	84
Scholl	25	2787	26	2680	27	2580	28	2488	29	2402
	30	2322	31	2247	32	2177	33	2111	34	2049
	35	1991	36	1935-39	37	1883	38	1834	39	1787-92
	40	1742-47	41	1699-706	42	1659-67	43	1621-27 ⁺	44	1584-94
	45	1549-60 ⁺	46	1515-22	47	1483-94	48	1452-63	49	1423-34 ⁺
	50	1394-403	51	1386	52	1386				

TABLE 6. Second Data Set for SALBP-2.

6. Benchmark Data Set for SALBP-G

We consider SALBP-G which is to minimize the sum of idle times $m \cdot c - t_{\text{sum}}$ and give a data set with 256 instances containing the precedence graphs of Table 1. Each instance is characterized by the name of the graph and a range $[\underline{m}, \bar{m}]$ of possible values of m . Optimal solutions are described by the parameters idle^* , m^* , and c^* denoting the minimal total idle time, the corresponding number of stations and the cycle time, respectively. If idle^* is not yet known, an interval is given. If several optimal combinations of m and c exist, one is arbitrarily chosen.

Name	\underline{m}	\bar{m}	idle^*	m^*	c^*	\underline{m}	\bar{m}	idle^*	m^*	c^*
Mertens	3	5	1	3	10	4	5	6	5	7
Bowman	3	5	9	3	28	4	5	10	5	17
Jaeschke	3	7	2	3	13	4	7	3	4	10
	5	7	8	5	9	6	7	11	6	8
Jackson	3	7	2	3	16	4	7	2	4	12
	5	7	4	5	10					
Mansoor	3	5	1	3	62	4	5	7	4	48
Mitchell	3	9	0	3	35	4	9	0	5	21
	6	9	3	6	18	7	9	7	8	14
Rosenberg	3	10	1	3	42	4	10	1	6	21
	7	10	3	8	16	9	10	15	10	14
Heskiaoff	3	10	0	4	256	5	10	1	5	205
	6	10	2	6	171	7	10	5	7	147
	8	10	8	8	129	9	10	20	9	116
Buxey	3	13	0	3	108	4	13	1	5	65
	6	13	4	8	41	6	7	5	7	47
	9	13	9	9	37	10	11	16	10	34
	10	13	12	12	28					
Sawyer	3	13	0	3	108	4	13	0	4	81
	5	13	1	5	65	6	13	4	8	41
	6	7	5	7	47	9	13	9	9	37
	10	11	16	10	34	10	13	12	12	28
Lutz 1	3	11	156	4	3574	5	11	220	5	2872
	6	11	236	6	2396	7	11	532	7	2096
	8	11	602	9	1638	10	11	1120	10	1526
Gunther	3	13	0	3	161	4	13	1	4	121
	5	13	2	5	97	6	13	3	9	54
	6	8	21	6	84	10	13	17	10	50
	11	13	45	11	48	12	13	45	12	44
Kilbrid	3	11	0	3	184	4	11	0	4	138
	5	11	0	6	92	7	11	0	8	69
	9	11	6	9	62	10	11	8	10	56
Hahn	3	4	335	3	4787	3	8	89	5	2823
	6	8	374	6	2400	7	8	1230	8	1907

Name	<u>m</u>	<u>m̄</u>	idle*	m*	c*	<u>m</u>	<u>m̄</u>	idle*	m*	c*
Warnecke	3	30	0	3	516	4	30	0	4	387
	5	30	0	6	258	7	30	0	9	172
	10	30	2	10	155	11	13	12	13	120
	11	30	6	14	111	15	30	12	15	104
	16	17	16	17	92	19	20	32	20	79
	19	30	27-32	20	79	22	30	36	24	66
	25	30	52-72	27	60	27	30	72	27	60
	28	30	76	29	56					
Tonge	3	23	0	3	1170	4	23	0	5	702
	6	23	0	6	585	7	23	2	8	439
	10	23	4	14	251	11	12	10	11	320
	14	23	4	14	251	16	17	26	17	208
	16	23	18	18	196	19	23	24	19	186
	20	23	30	20	177	21	23	54	22	162
Wee-Mag	3	38	1	3	500	4	38	1	4	375
	5	38	1	5	300	6	38	1	6	250
	7	38	1	10	150	7	9	4	9	167
	11	38	1	12	125	13	14	9	13	116
	13	38	1	15	100	16	18	5	16	94
	16	19	5	16	94	16	38	5	16	94
	21	24	13	21	72	21	38	5-13	21	72
	22	24	19	22	69	26	29	191	26	65
	26	31	113	31	52	26	38	5-37	32	48
	27	29	229-256	27	65	32	38	5-37	32	48
35	38	111	35	46	36	38	157	36	46	
Arcus 1	3	21	1	3	25236	4	21	1	4	18927
	5	21	3	5	15142	6	21	13	6	12620
	7	21	75	7	10826	8	21	93	10	7580
	8	9	725	8	9554	11	12	1237	12	6412
	11	13	525	13	5864	11	21	467	14	5441
	15	17	853	15	5104	15	21	853	15	5104
	16	17	1065	17	4516	19	21	1585	19	4068
Lutz 2	3	49	1	3	162	4	49	1	6	81
	7	49	1	9	54	10	49	5	10	49
	11	13	7	12	41	11	49	5	14	35
	15	16	10	15	33	15	49	8	17	29
	18	28	9	19	26	18	49	8	29	17
	20	25	15	20	25	20	26	9	26	19
	21	24	19	21	24	21	25	15	25	20
	22	24	19	24	21	30	34	11	31	16
	30	49	11	31	16	32	34	25	34	15
	36	49	33	37	14	42	44	43	44	12
	42	49	43	44	12	46	48	67	46	12
Lutz 3	3	23	0	3	548	4	23	0	4	411
	5	23	1	5	329	6	23	6	10	165
	7	9	8	7	236	8	9	12	9	184
	11	23	6	15	110	12	14	8	14	118
	12	15	6	15	110	17	23	22	17	98
	18	21	30	18	93	18	23	28	22	76
	19	20	36-47	19	89	19	21	36	21	80

Name	<u>m</u>	<u>m̄</u>	idle*	m*	c*	<u>m</u>	<u>m̄</u>	idle*	m*	c*
Arcus 2	3	27	0	3	50133	4	27	0	9	16711
	4	8	1	4	37600	5	8	1	5	30080
	6	7	3	6	25067	6	8	1	8	18800
	10	16	1-4	11	13673	10	27	1-4	11	13673
	11	16	4	11	13673	12	16	9	12	12534
	13	16	11	13	11570	14	16	59	14	10747
	15	16	126-141	15	10036	18	19	81-233	19	7928
	18	27	1-121	20	7526	21	24	59-499	22	6859
	21	27	59-499	22	6859	22	23	59-499	22	6859
	22	24	59-499	22	6859	26	27	1545-1857	26	5856
	Bart- holdi 1	3	15	0	3	1878	4	15	0	6
4		5	1	5	1127	7	15	0	9	626
7		8	1	7	805	10	15	6	10	564
11		15	6	12	470	13	15	8	13	434
14		15	8	14	403					
Bart- holdi 2	3	28	1	5	847	3	52	0	29	146
	6	28	1	7	605	8	10	5	9	471
	8	28	1	11	385	12	28	2	12	353
	13	18	4	13	326	13	28	3	19	223
	14	18	6	16	265	20	25	6	20	212
	20	28	4	26	163	21	25	8	21	202
	22	25	12	22	193	23	25	14	24	177
	27	28	5	27	157	30	34	13	31	137
	30	52	1	35	121	32	33	22	32	133
	32	34	16	34	125	36	39	14	36	118
	36	52	6	40	106	37	39	17	39	109
	41	52	8	42	101	43	49	23	43	99
	43	52	16-23	43	99	44	48	34	44	97
	44	49	29	49	87	45	47	41	45	95
45	48	38	48	89	51	52	50	51	84	
Scholl	3	4	1	4	17414	3	51	0	5	13931
	6	51	1	8	8707	6	7	2	7	9951
	9	20	5	9	7740	9	51	2	21	3317
	10	20	5	10	6966	11	20	5	12	5805
	13	14	9	14	4976	13	20	5	15	4644
	16	17	9	16	435	16	20	5	18	3870
	19	20	5	20	3483	22	26	12	23	3029
	22	28	5	27	2580	22	30	3	29	2402
	22	51	2	31	2247	24	26	17	24	2903
	25	26	20	25	2787	32	35	8	33	2111
	32	40	5-8	33	2111	32	51	4-8	33	2111
	34	35	11	34	2049	37	40	16	37	1883
	38	39	37	38	1834	38	40	25-37	38	1834
	42	51	23-306	43	1627	44	45	41-481	44	1594
	44	48	35-481	44	1594	44	51	35-481	44	1594
	47	48	41-563	47	1494	50	51	45-495	50	1403

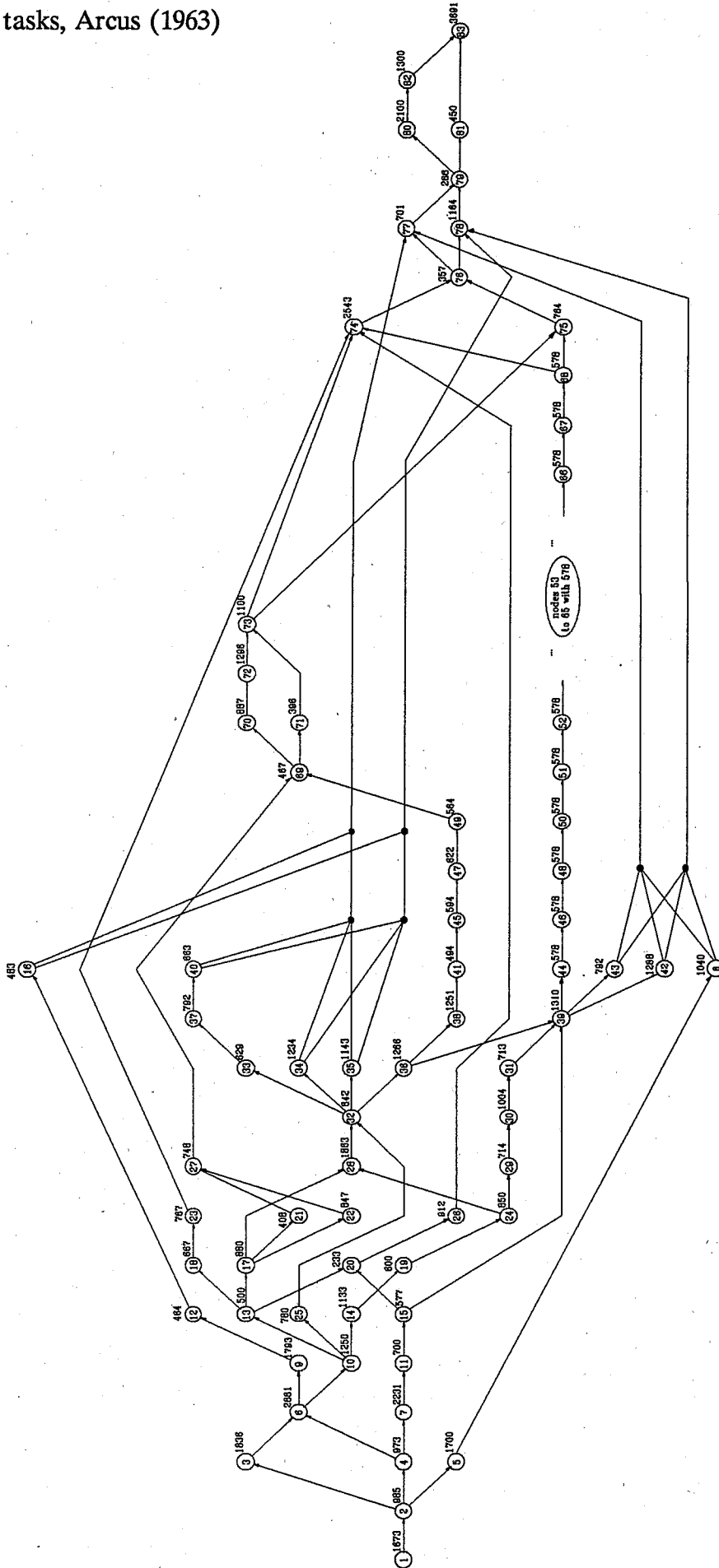
References

- Arcus, A.L.* (1963): An analysis of a computer method of sequencing assembly line operations. Ph.D. dissertation, University of California, Berkely.
- Baker, K.R.* (1974): Introduction to sequencing and scheduling. Wiley, New York.
- Bartholdi, J.J.* (1993): Balancing two-sided assembly lines: a case study. *International Journal of Production Research* 31, 2447 - 2461.
- Baybars, I.* (1986): A survey of exact algorithms for the simple assembly line balancing problem. *Management Science* 32, 909 - 932.
- Bhattacharjee, T.K. and S. Sahu* (1990): Complexity of single model assembly line balancing problems. *Engineering Costs and Production Economics* 18, 203 - 214.
- Bowman, E.H.* (1960): Assembly-line balancing by linear programming. *Operations Research* 8, 385 - 389.
- Buxey, G.M.* (1974): Assembly line balancing with multiple stations. *Management Science* 20, 1010 - 1021.
- Dar-El, E.M.* (1973): MALB - a heuristic technique for balancing large single-model assembly lines. *AIIE Transactions* 5, 343 - 356.
- Domschke, W.; A. Scholl and S. Voß* (1993): *Produktionsplanung - Ablauforganisatorische Aspekte*. Springer, Berlin.
- Gunther, R.E.; G.D. Johnson and R.S. Peterson* (1983): Currently practiced formulations for the assembly line balance problem. *Journal of Operations Management* 3, 209 - 221.
- Hahn, R.* (1972): *Produktionsplanung bei Linienfertigung*. de Gruyter, Berlin.
- Heskiaoff, H.* (1968): A heuristic method for balancing assembly lines. *The Western Electric Engineer* 12/3, 9 - 16.
- Hoffmann, T.R.* (1990): Assembly line balancing: a set of challenging problems. *International Journal of Production Research* 28, 1807 - 1815.
- Hoffmann, T.R.* (1992): EUREKA: a hybrid system for assembly line balancing. *Management Science* 38, 39 - 47.
- Jackson, J.R.* (1956): A computing procedure for a line balancing problem. *Management Science* 2, 261 - 271.
- Jaeschke, G.* (1964): Branching and Bounding: Eine allgemeine Methode zur Lösung kombinatorischer Probleme. *Ablauf- und Planungsforschung* 5, 133 - 155.
- Johnson, R.V.* (1988): Optimally balancing large assembly lines with 'FABLE'. *Management Science* 34, 240 - 253.
- Karp, R.M.* (1972): Reducibility among combinatorial problems. In: Miller, R.E. and J.W. Thatcher (eds.): *Complexity of computer computation*, Plenum Press, New York, 85 - 103.
- Kilbridge, M.D. and L. Wester* (1962): A review of analytical systems of line balancing. *Operations Research* 10, 626 - 638.
- Lutz, L.* (1974): *Abtakte von Montagelinien*. Krausskopf, Mainz.
- Mansoor, E.M.* (1964): Assembly line balancing - an improvement on the ranked positional weight technique. *Journal of Industrial Engineering* 15, 73 - 77 and 322 - 323.
- Mastor, A.A.* (1970): An experimental investigation and comparative evaluation of production line balancing techniques. *Management Science* 16, 728 - 746.
- Mertens, P.* (1967): Fließbandabstimmung mit dem Verfahren der begrenzten Enumeration nach Müller-Merbach. *Ablauf- und Planungsforschung* 8, 429 - 433.
- Mitchell, J.* (1957): A computational procedure for balancing zoned assembly lines. Research Report 6-94801-1-R3, Westinghouse Research Laboratories, Pittsburgh.

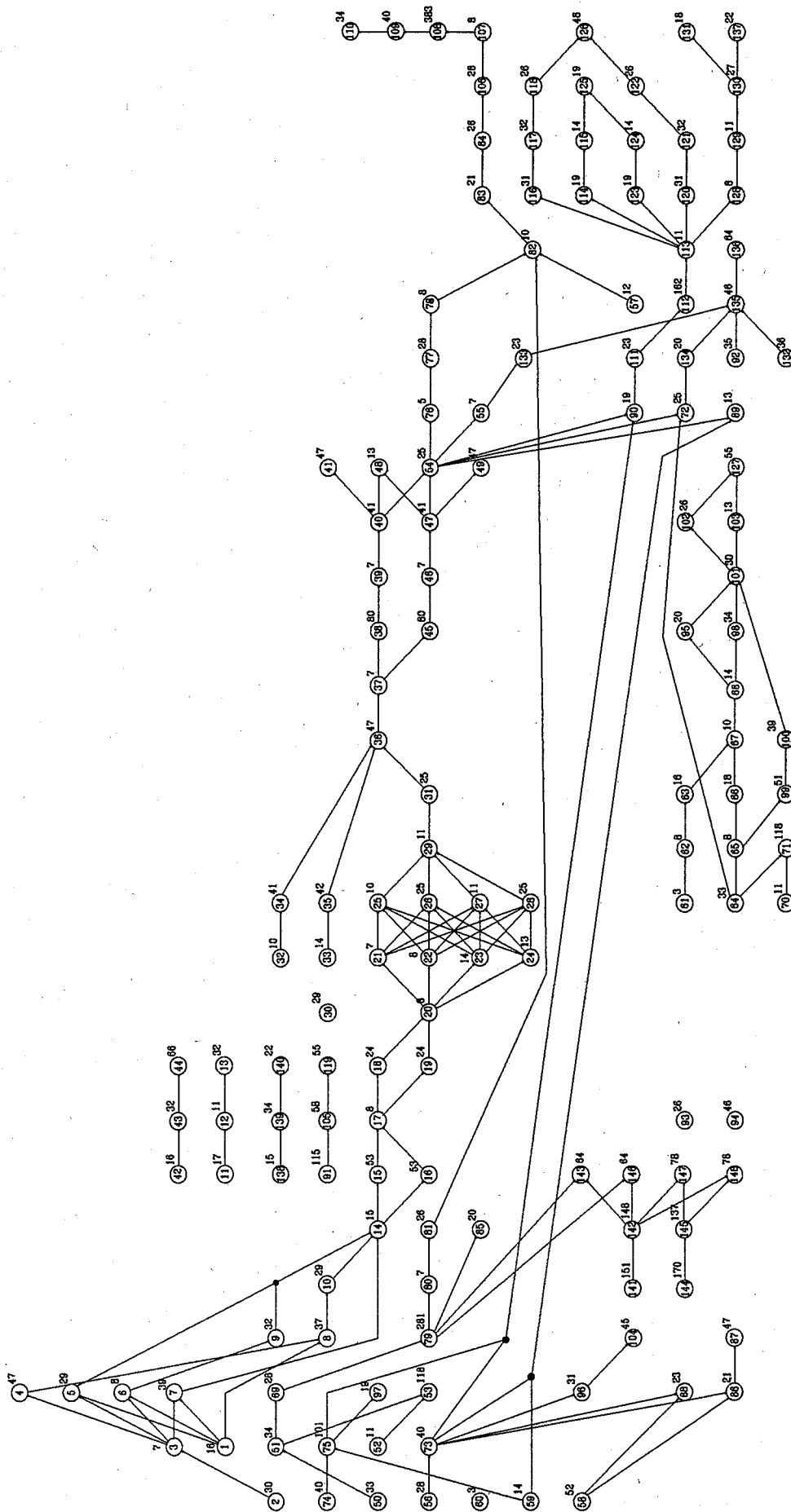
- Mukherjee, S.K. and S.K. Basu* (1964): An application of heuristic method of assembly line balancing in an Indian industry. *Proc. Instn. Mech. Engrs.* 178/1/11, 277 - 292.
- Rosenberg, O. and H. Ziegler* (1992): A comparison of heuristic algorithms for cost-oriented assembly line balancing. *Zeitschrift für Operations Research* 36, 477 - 495.
- Sawyer, J.H.F.* (1970): *Line balancing*. The Machinery Publishing, Brighton.
- Talbot, F.B; J.H. Patterson and W.V. Gehrlein* (1986): A comparative evaluation of heuristic line balancing techniques. *Management Science* 32, 430 - 454.
- Tonge, F.M.* (1960): Summary of a heuristic line balancing procedure. *Management Science* 7, 21 - 39.
- Tonge, F.M.* (1961): A heuristic program for assembly line balancing. Prentice Hall Inc., Englewood Cliffs.
- Warnecke, H.J.* (1971): Anwendung mathematischer Methoden bei der Leistungsabstimmung von Montagelinien. *CIRP - Annals* 20, 99 - 100.
- Wee, T.S. and M.J. Magazine* (1981): An efficient branch and bound algorithm for an assembly line balancing problem - part II: maximize the production rate. Working Paper No. 151, University of Waterloo, Waterloo.

Appendix: Precedence Graphs

Arcus 1: 83 tasks, Arcus (1963)



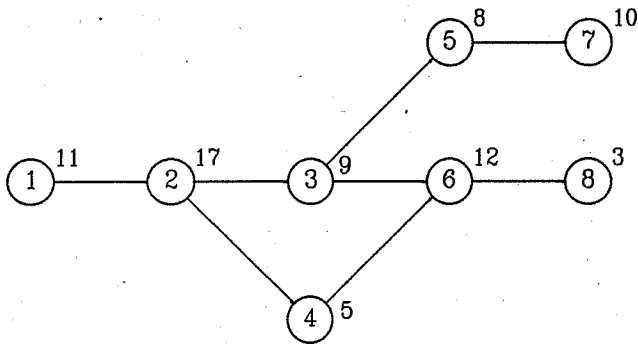
Bartholdi 1: 148 tasks, assembly of small utility vehicles, Bartholdi (1993), renumbered topologically



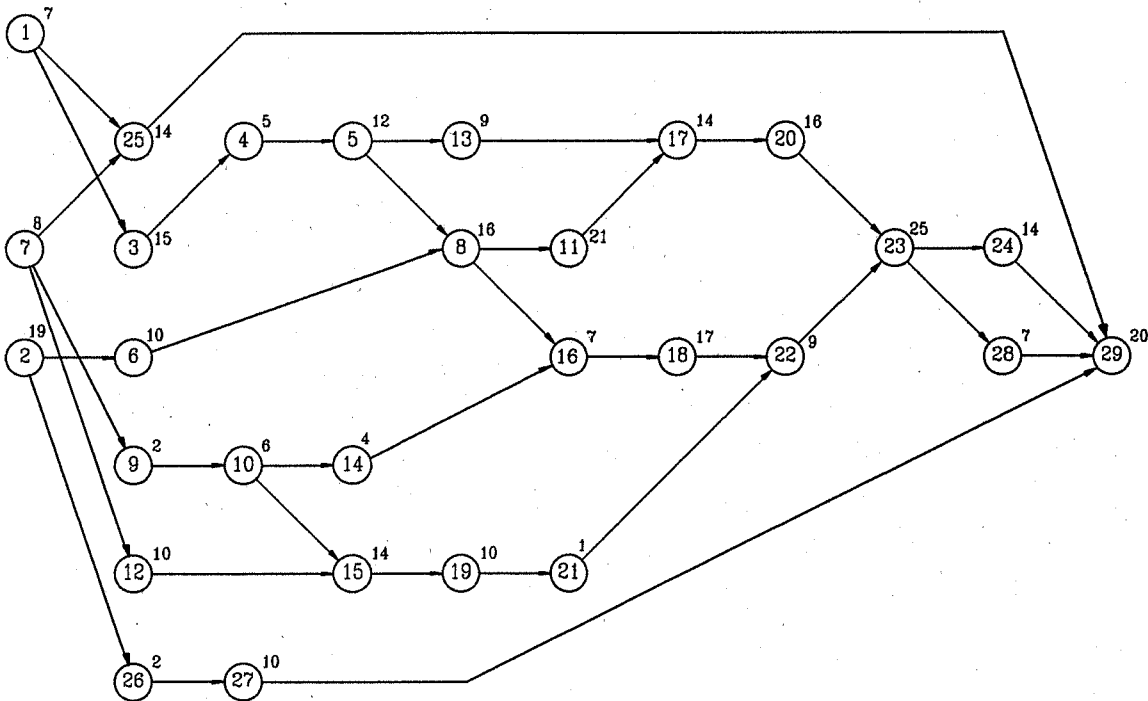
Bartholdi 2: 148 tasks, Bartholdi (1993), modified operation times

It differs from Bartholdi 1 only with respect to the operation times. In order to complicate the problem, the operation times are computed by $t_j \bmod 100$ (the last two digits of the original times are used).

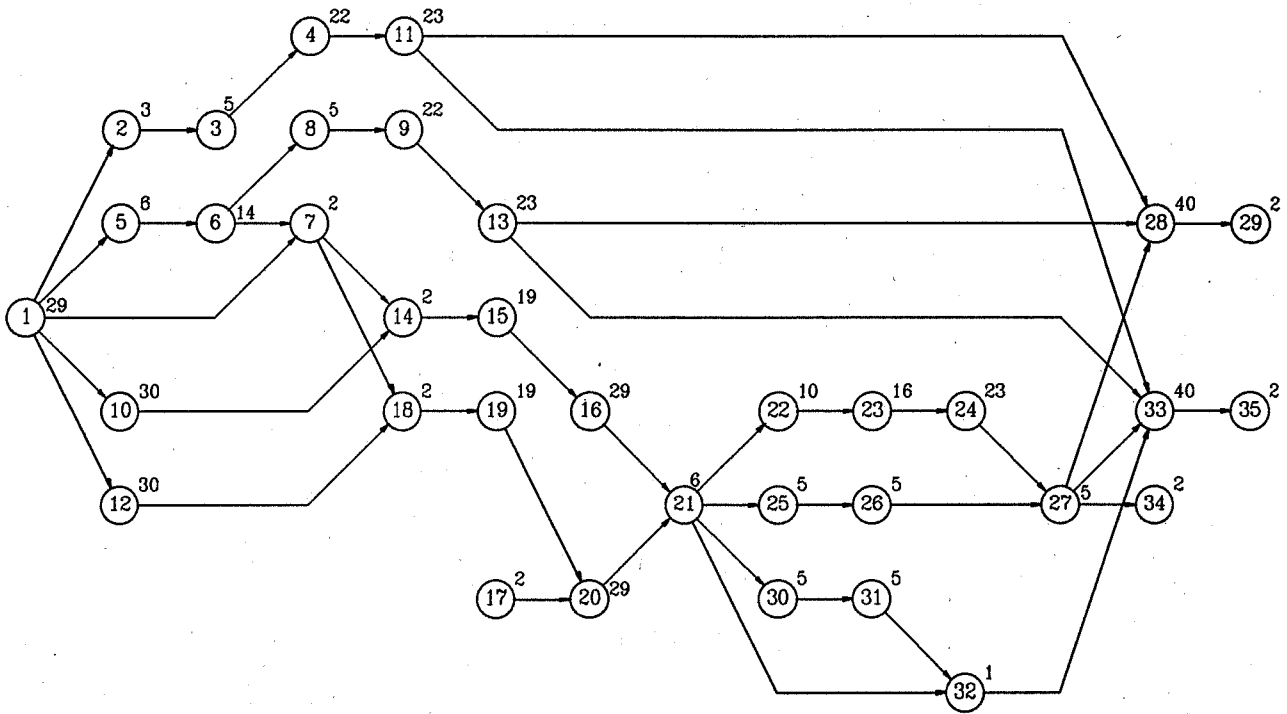
Bowman: 8 tasks, Bowman (1960)



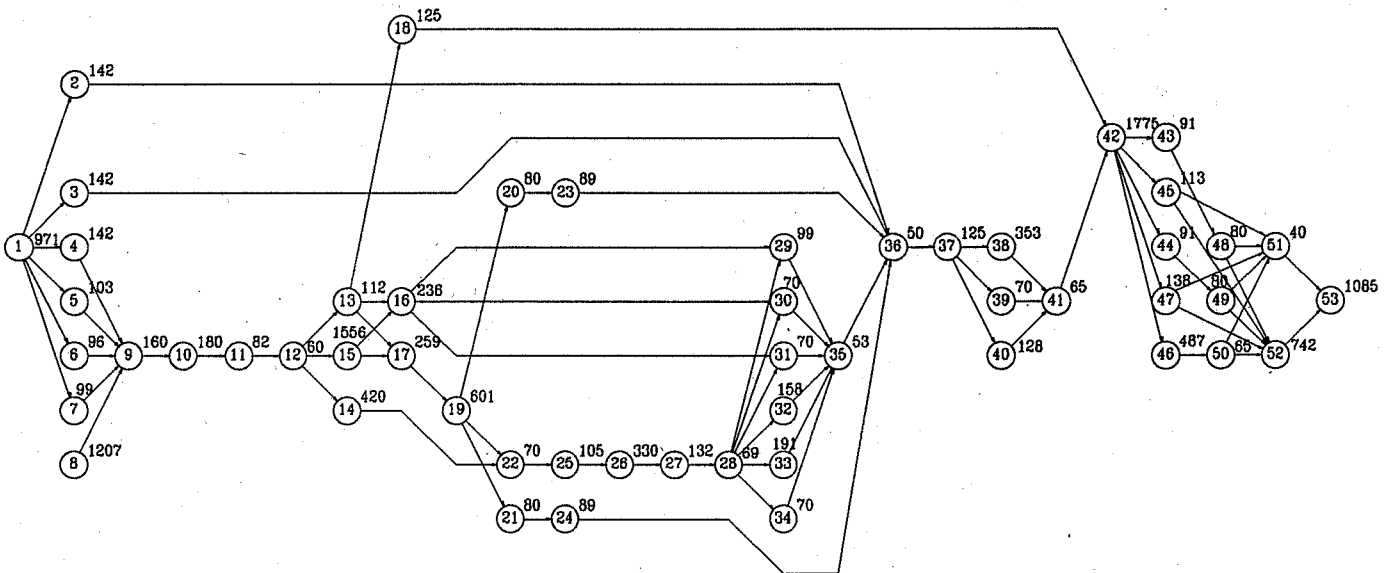
Buxey: 29 tasks, Buxey (1974), modified version of Sawyer (see below)



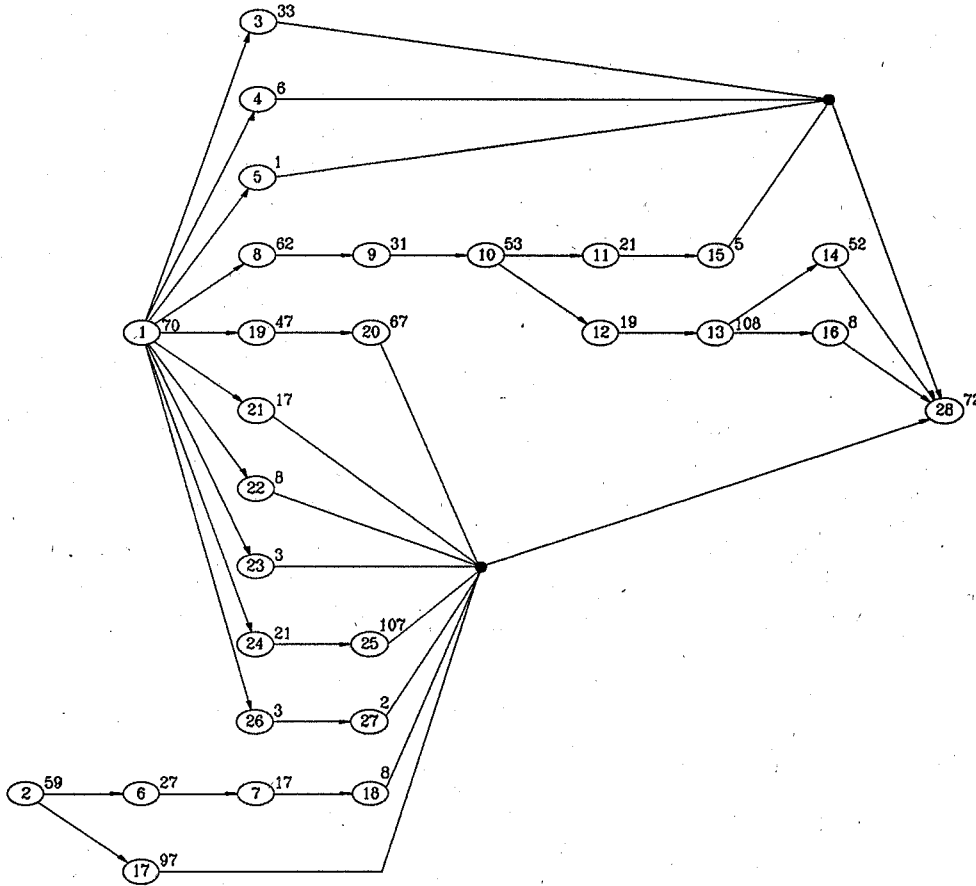
Gunther: 35 tasks, assembly of an auto engine cradle, Gunther et al. (1983)



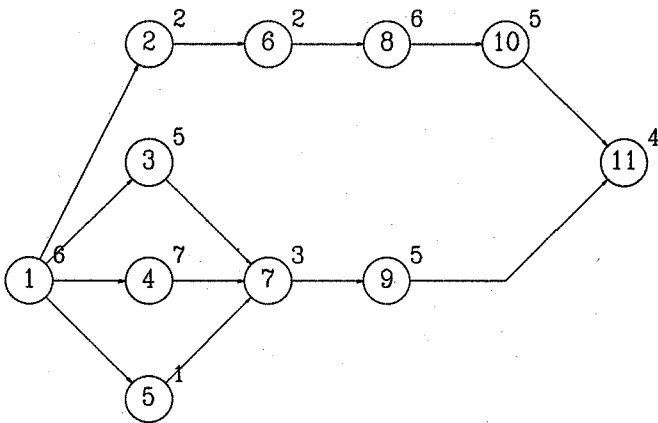
Hahn: 53 tasks (without cycle-independent tasks), assembly of a hot water tank, Hahn (1972, pp. 47)



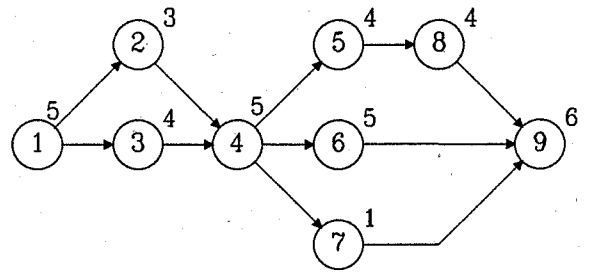
Heskiaoff: 28 tasks, reconditioning of telephone sets, Heskiaoff (1968), modified as in the data set of Talbot et al. (the arc (1,2) was originally contained)



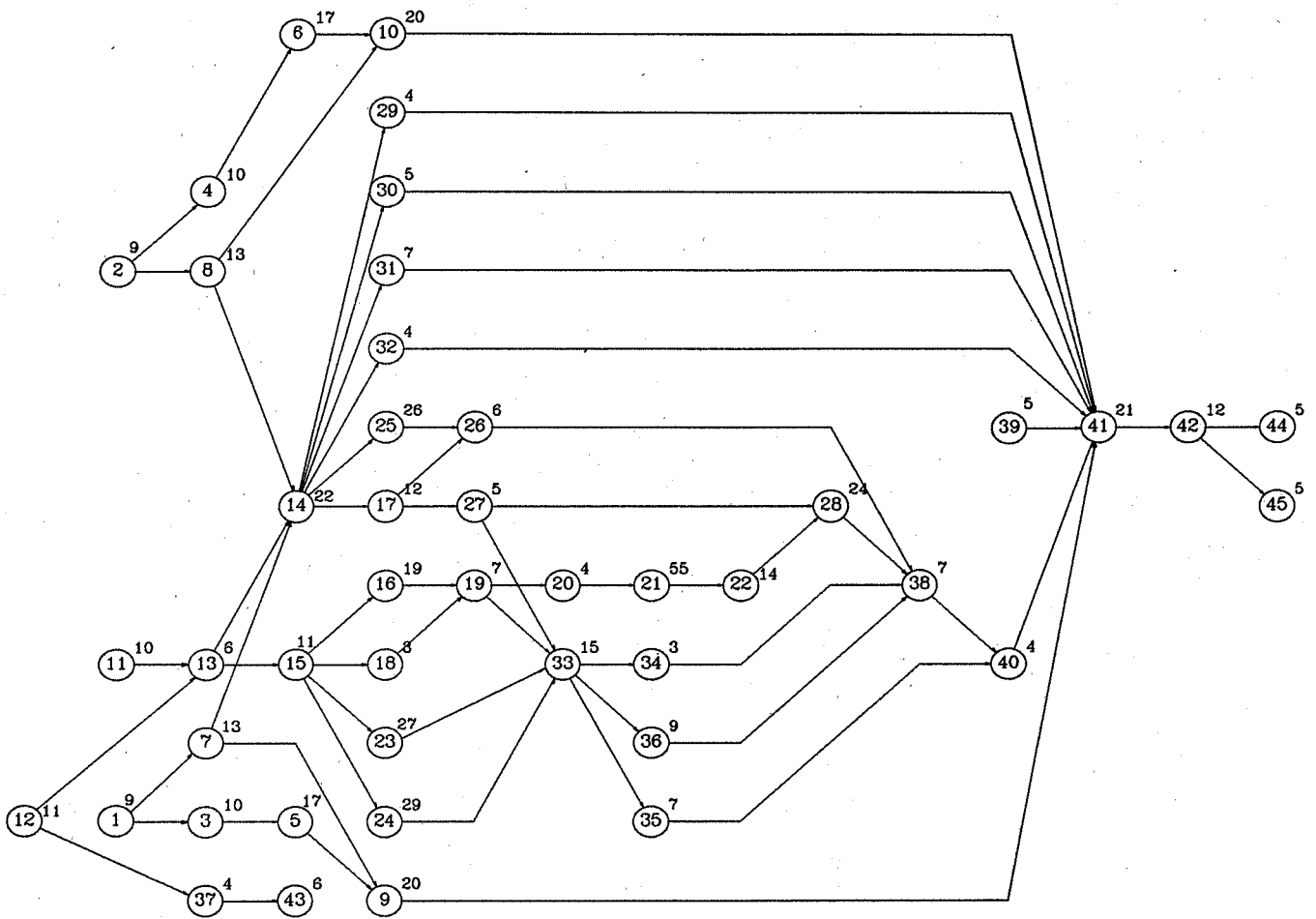
Jackson: 11 tasks, Jackson (1956)



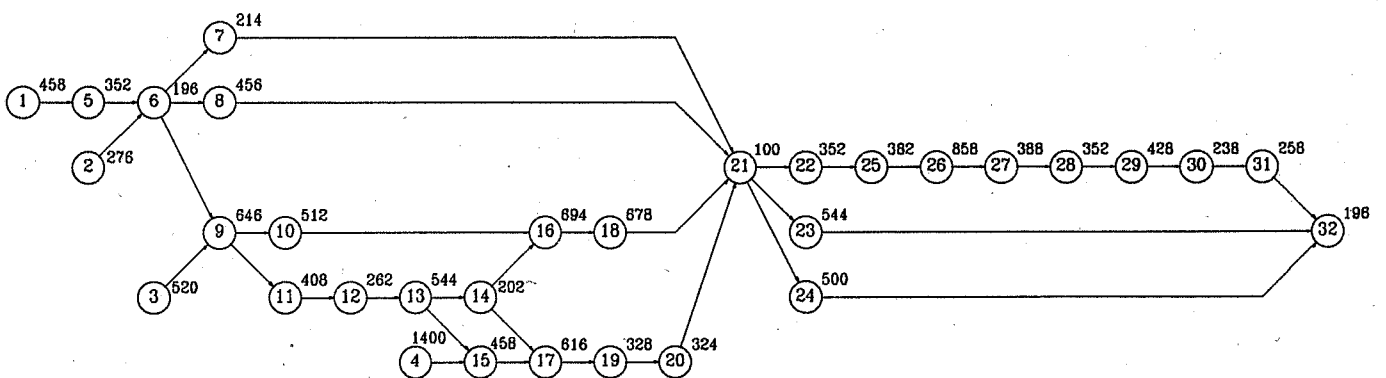
Jaeschke: 9 tasks, Jaeschke (1964)



Kilbrid: 45 tasks, Kilbridge and Wester (1962)



Lutz 1: 32 tasks, assembly of a pressure reducing device, Lutz (1974, pp. 89-90)

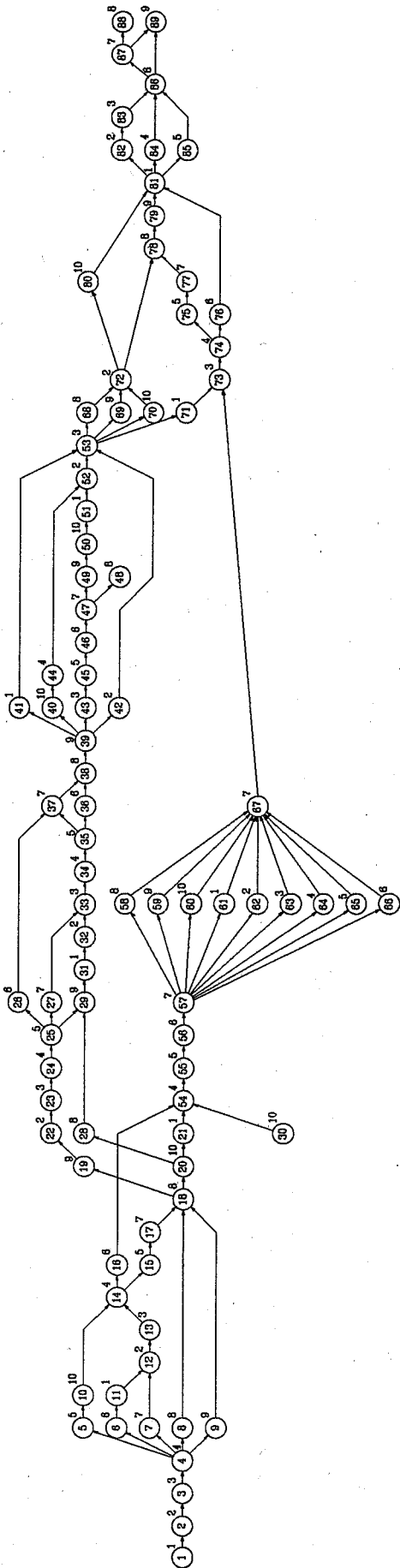


Lutz 2: 89 tasks, modification of Lutz 3 with operation times $t_j := j \bmod 10$, tasks with $t_j = 0$ get the value $t_j = 10$

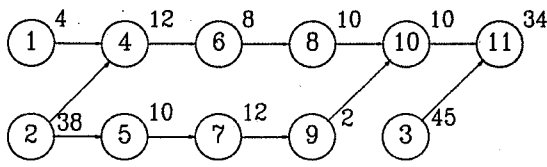
Lutz 3: 89 tasks, assembly of a refrigerator, Lutz (1974, p. 118–119), precedence constraints as Lutz 2

Task times:

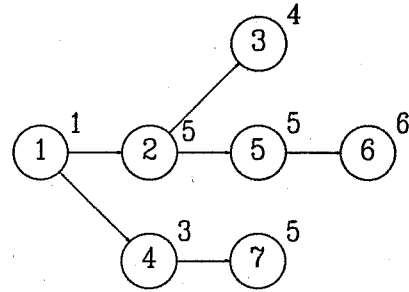
j	t_j	t_{j+1}	t_{j+2}	t_{j+3}	t_{j+4}
1	28	17	7	1	1
6	16	13	8	13	24
11	16	24	12	20	22
16	22	22	47	6	12
21	6	22	7	14	68
26	15	8	74	32	11
31	18	15	42	27	38
36	38	26	35	31	8
41	14	8	8	8	17
46	7	19	12	24	26
51	22	14	27	60	4
56	2	1	9	5	6
61	7	12	2	11	14
66	6	21	14	1	6
71	21	11	14	28	12
76	11	30	22	14	31
81	18	16	7	38	8
86	31	29	12	18	



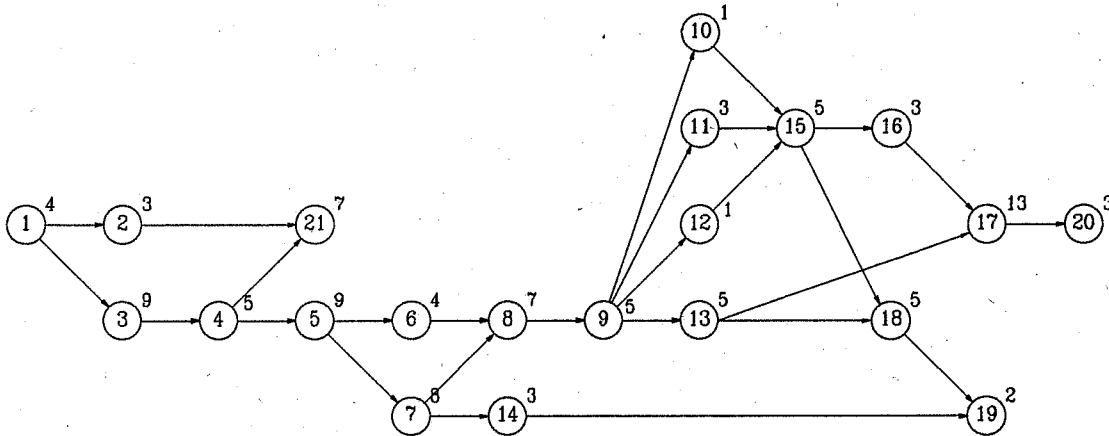
Mansoor: 11 tasks, Mansoor (1964)



Mertens: 7 tasks, Mertens (1967)



Mitchell: 21 tasks, Mitchell (1957), Tonge (1960)



Mukherjee: 94 tasks, assembly of an electronic consumer good in an Indian manufactory, Mukherjee and Basu (1964)

Task times:

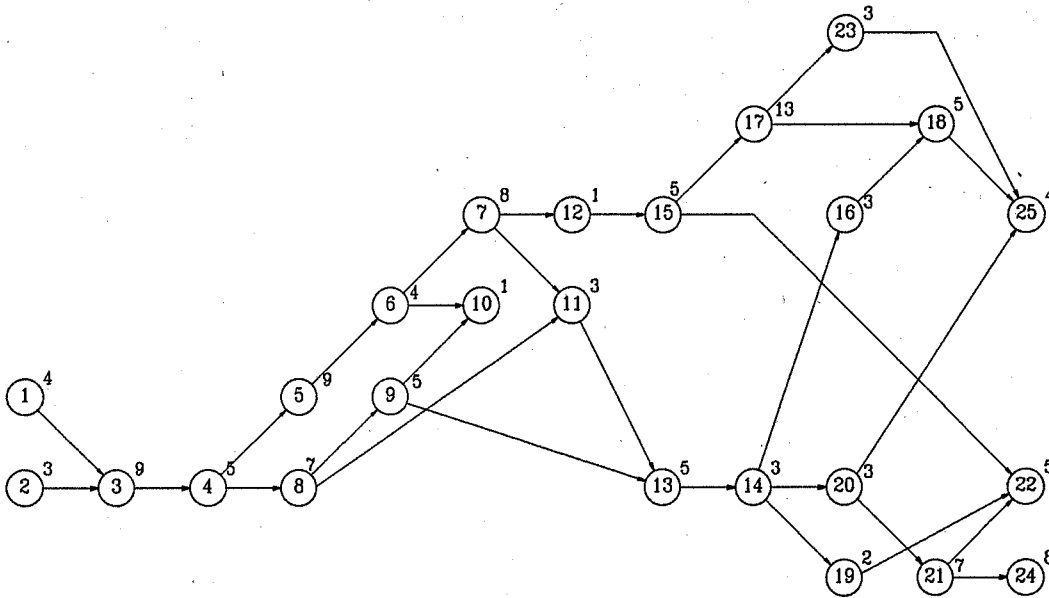
j	t_j	t_{j+1}	t_{j+2}	t_{j+3}	t_{j+4}	t_{j+5}	t_{j+6}	t_{j+7}	t_{j+8}	t_{j+9}
1	158	42	17	24	71	66	50	55	30	28
11	113	114	57	65	15	11	10	19	8	17
21	57	40	17	11	17	27	17	18	16	18
31	50	18	13	21	63	8	63	9	20	72
41	30	69	27	9	41	38	76	31	11	29
51	48	8	171	48	43	18	94	9	19	24
61	8	34	62	12	8	47	27	18	52	65
71	72	51	58	53	76	93	86	76	9	14
81	55	32	22	35	63	59	149	40	24	123
91	25	115	103	54						

Precedence constraints: pairs i, j for task i preceding task j

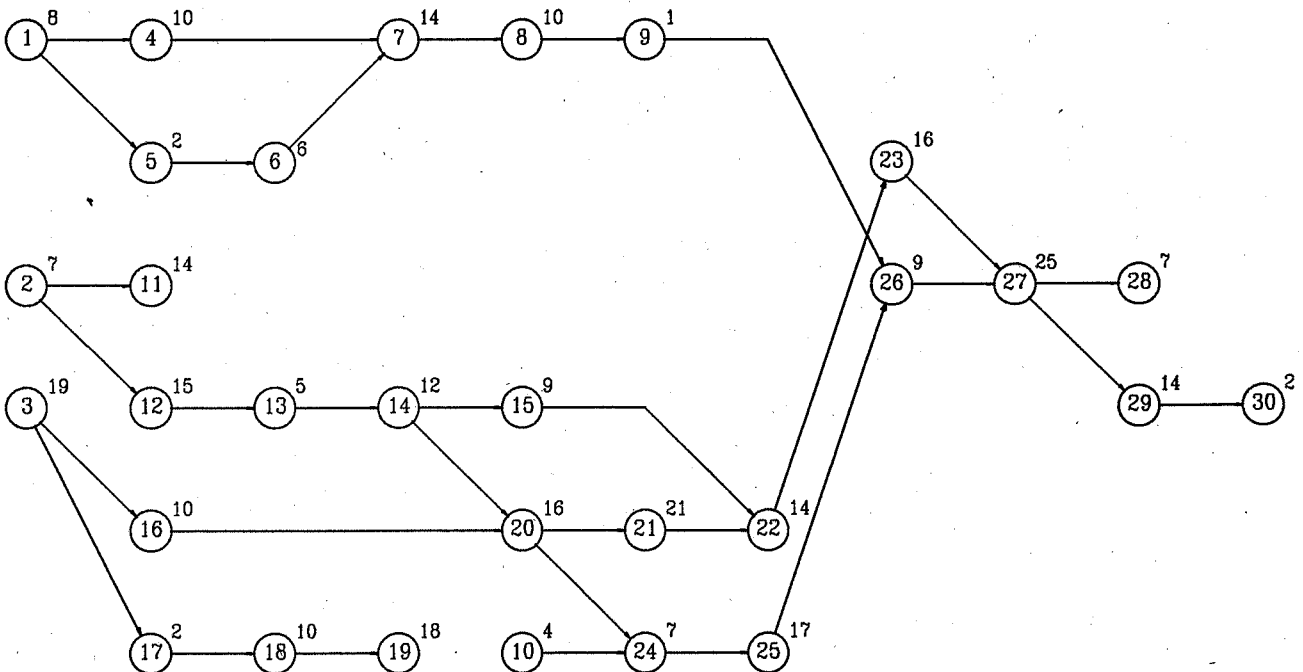
1,2	1,3	1,4	1,5	1,6	1,7	1,8	1,9
1,10	2,11	2,17	3,12	4,11	4,16	5,11	5,13
5,18	5,16	6,11	6,15	6,13	6,19	7,42	8,29
9,11	10,19	10,37	10,48	10,50	11,14	12,20	12,61
12,52	13,82	14,25	14,23	14,30	14,31	14,24	14,32
14,22	14,21	14,26	14,28	14,27	14,29	15,81	16,75
17,82	18,75	19,75	20,78	21,71	21,60	21,53	21,39
21,40	21,41	22,38	22,72	22,49	22,58	23,34	23,35
24,37	24,44	25,57	25,63	25,33	25,73	25,60	25,65
25,56	26,41	26,43	27,62	27,58	27,54	27,55	27,56

28,33	28,35	28,40	28,43	28,49	28,51	28,52	28,54
29,75	30,59	30,42	30,36	31,74	32,36	32,47	32,44
32,60	32,45	32,69	32,46	32,48	32,50	33,78	34,78
35,78	36,82	37,61	37,73	37,64	37,66	37,67	37,68
37,69	37,70	37,71	37,72	37,62	38,82	39,82	40,78
41,78	42,74	43,78	44,75	45,75	46,75	46,76	47,75
48,75	49,78	50,75	51,78	52,78	53,76	53,77	54,78
55,77	56,77	57,76	57,77	58,77	59,82	60,77	61,77
62,77	63,77	64,74	65,74	66,75	66,76	67,74	68,82
69,77	69,75	70,77	70,76	71,76	72,76	73,77	74,82
75,82	76,82	77,78	78,79	78,80	79,81	80,81	81,82
82,83	83,84	83,85	84,86	85,86	86,87	87,88	88,89
88,90	89,92	90,91	90,93	92,94			

Rosenberg: 25 tasks, randomly generated, Rosenberg and Ziegler (1992)



Sawyer: 30 tasks, Sawyer (1970, p. 33), modified as in the data set of Talbot et al. (the arcs (1,11) and (6,10) were originally contained)



Scholl: 297 tasks, assembly of an engine

Task times:

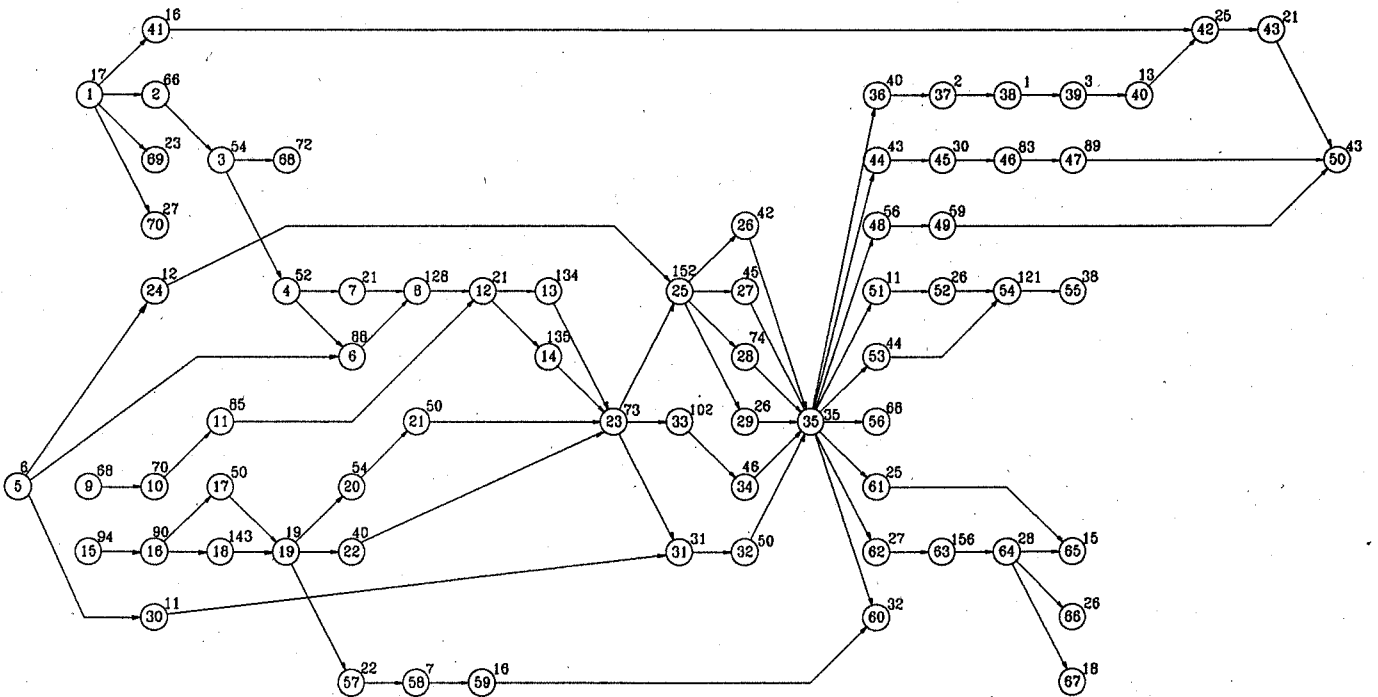
j	t_j	t_{j+1}	t_{j+2}	t_{j+3}	t_{j+4}	t_{j+5}	t_{j+6}	t_{j+7}	t_{j+8}	t_{j+9}
1	270	270	130	148	190	293	348	182	490	212
11	248	248	248	248	248	268	268	268	288	248
21	268	60	268	240	240	171	490	182	170	306
31	108	248	190	240	339	288	248	455	268	270
41	180	121	270	440	249	194	162	130	388	90
51	212	246	188	270	160	79	466	240	137	184
61	110	275	149	280	119	184	140	150	190	150
71	150	284	192	347	232	140	608	80	40	130
81	110	350	140	240	240	90	54	294	203	150
91	270	155	190	78	140	241	430	90	110	9
101	430	130	289	110	160	442	159	250	190	184
111	690	72	190	190	90	889	170	155	190	130
121	390	301	54	227	142	184	741	868	230	121
131	320	126	440	127	134	150	140	110	320	250
141	232	188	250	377	90	140	90	90	70	90
151	110	150	101	377	118	290	209	150	150	79
161	150	91	59	218	351	873	130	68	126	120
171	227	198	132	121	150	100	38	70	355	284
181	122	75	160	140	520	99	182	80	514	96
191	50	272	226	194	164	96	107	108	167	98
201	82	482	72	50	130	230	50	240	190	190
211	240	74	139	339	260	132	550	420	152	12
221	90	5	128	100	120	100	320	835	740	223
231	100	390	140	304	120	403	21	246	160	1019
241	34	120	68	910	302	778	101	1310	20	278
251	81	290	100	372	72	28	90	250	144	303
261	220	58	224	211	99	44	120	70	421	231
271	214	196	280	398	72	280	356	193	140	130
281	300	456	7	170	252	210	308	308	121	52
291	426	104	1386	527	968	1047	538			

Precedence constraints: pairs i, j for task i preceding task j

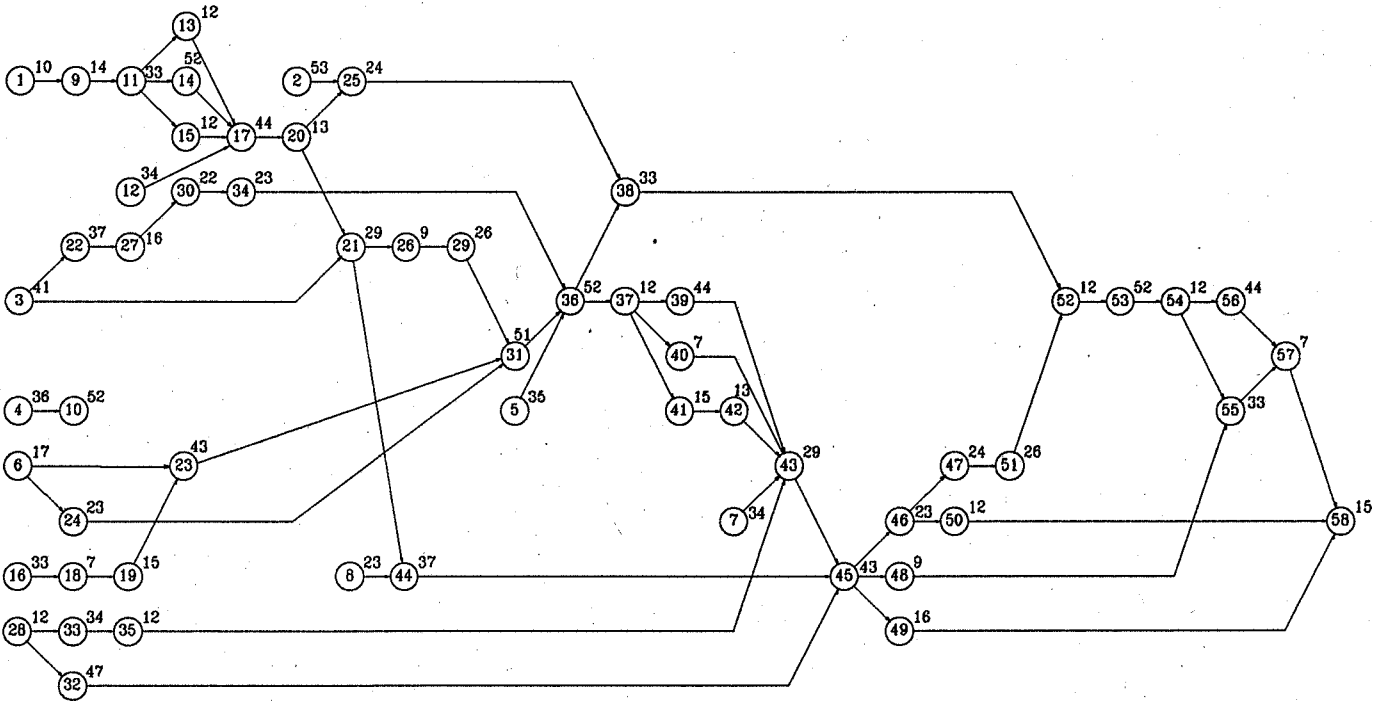
1,2	2,3	3,4	4,5	4,22	4,26	4,27	4,40
4,48	4,56	4,83	4,86	4,94	4,105	4,109	4,111
4,134	4,221	4,247	4,259	5,6	6,7	6,8	6,9
6,10	7,11	7,12	7,13	7,14	7,15	7,20	8,11
8,12	8,13	8,14	8,15	8,20	9,11	9,12	9,13
9,14	9,15	9,20	10,11	10,12	10,13	10,14	10,15
10,20	11,16	12,17	13,18	14,19	15,21	16,23	17,23
18,23	19,23	20,23	21,23	22,24	22,25	23,28	24,29
25,29	26,30	27,31	28,32	28,37	29,33	29,44	29,121
30,34	30,297	31,34	31,82	31,172	31,179	32,36	33,38
34,35	35,42	36,39	37,39	38,41	39,43	40,44	40,84
40,97	41,45	42,46	43,47	44,49	45,50	46,51	46,138
47,52	48,52	49,53	50,54	51,55	51,81	52,57	53,58
54,58	54,296	55,59	56,60	56,61	57,62	57,63	57,71
57,76	58,64	59,64	59,99	59,100	60,68	61,65	62,66
63,67	64,72	65,69	66,69	67,70	68,73	69,74	70,75
71,77	72,78	73,84	73,97	74,84	74,97	75,84	75,97
76,84	76,97	77,84	77,97	78,79	78,80	78,125	78,192
79,85	80,85	81,87	82,88	82,89	83,90	84,91	85,92
86,93	87,99	87,100	88,99	88,100	89,99	89,100	90,95
91,96	92,98	93,98	94,101	95,101	96,101	97,101	98,102

99,103	100,104	101,106	102,107	103,108	104,108	105,110	106,112
107,113	108,114	108,115	108,292	109,119	109,120	110,119	110,120
110,162	111,116	112,117	113,118	114,119	115,120	116,122	117,123
117,124	117,257	118,126	119,127	120,127	120,150	121,128	122,129
123,130	123,145	123,146	123,147	123,148	123,149	124,130	125,130
126,130	127,130	127,157	128,130	129,130	129,141	130,131	130,144
131,132	131,133	132,135	133,135	133,170	134,136	135,137	136,139
137,140	138,140	138,191	139,142	139,253	140,143	140,200	141,151
142,152	143,153	143,169	144,154	145,155	146,156	147,158	148,159
149,160	150,161	151,163	152,164	153,165	154,166	155,166	156,166
157,166	158,166	159,166	160,166	161,166	162,167	163,166	164,167
165,168	165,176	166,170	167,171	168,173	169,174	170,174	171,174
172,175	173,177	174,178	174,287	174,288	175,180	176,181	177,181
178,181	179,181	180,181	180,252	181,182	181,183	181,184	181,185
181,186	181,187	181,188	181,189	181,196	181,197	181,295	182,190
183,193	184,194	185,195	186,195	187,195	188,195	189,195	190,195
191,200	192,201	193,198	194,199	195,199	195,203	195,205	195,227
195,229	196,202	197,202	198,202	199,202	200,202	201,202	202,204
202,251	203,206	203,208	204,207	204,250	205,207	206,209	207,210
207,212	208,210	209,210	209,211	210,213	211,213	212,214	213,214
214,215	214,234	215,216	216,217	217,218	218,219	219,220	220,222
221,223	222,224	223,225	224,226	225,227	226,228	227,230	228,231
229,235	229,236	230,232	230,271	230,289	231,233	232,236	233,237
234,238	234,256	235,237	236,239	237,240	238,240	238,285	239,240
239,279	240,241	240,243	241,242	242,244	243,245	243,246	244,245
244,246	244,255	245,248	246,248	247,278	248,249	249,254	249,284
250,256	251,256	252,258	253,260	254,261	255,261	255,262	256,263
257,264	258,265	259,266	260,267	261,268	261,269	262,269	263,270
264,271	265,272	266,271	267,273	268,274	269,274	270,274	271,274
272,275	273,276	274,277	274,278	274,282	275,280	276,281	277,283
278,283	279,286	280,290	281,291	282,293	283,293	284,294	285,294
286,294	287,293	288,293	289,294	290,293	291,293	292,293	

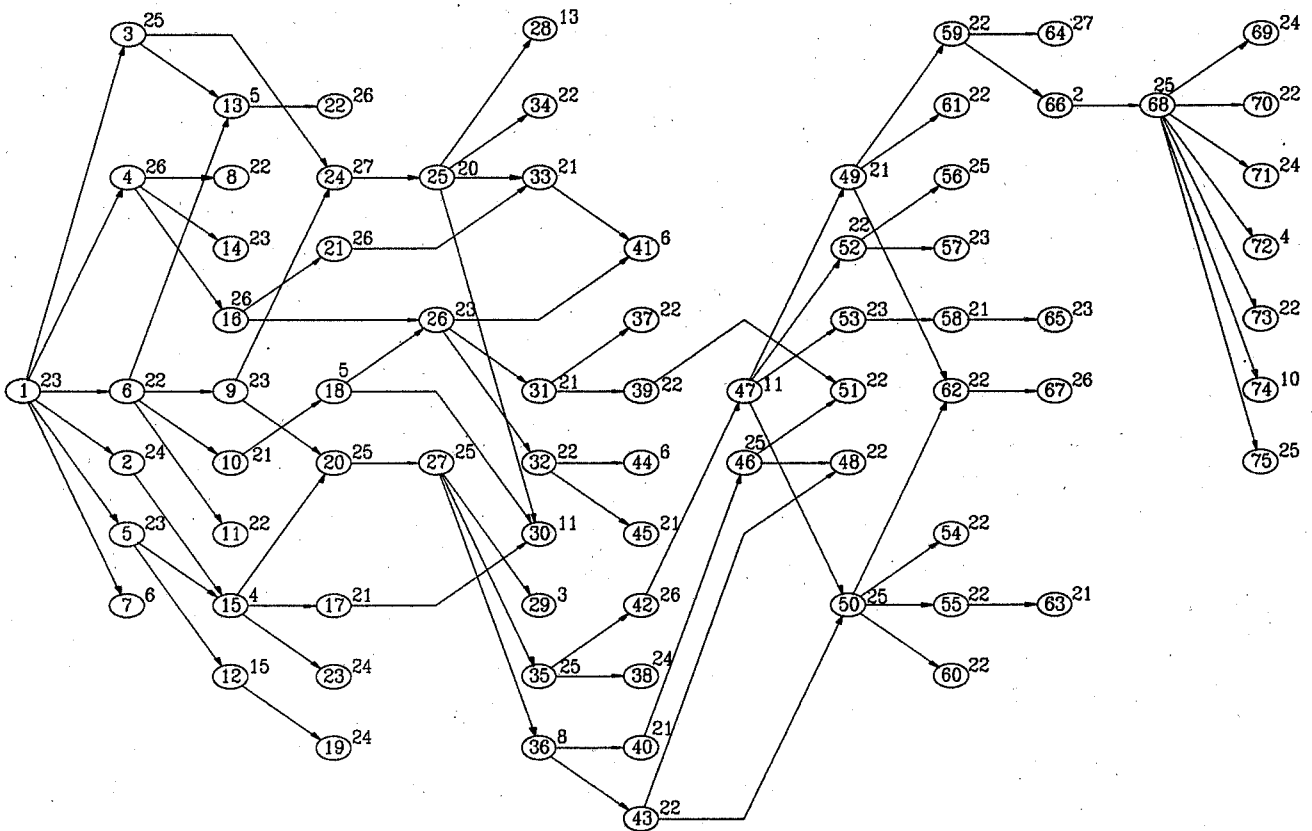
Tonge: 70 tasks, electronics industry, Tonge (1961)



Warnecke: 58 tasks, assembly of a miniature camera, Warnecke (1971), operation times randomly generated, because they are not reported in the reference



Wee-Mag: 75 tasks, randomly generated, Wee and Magazine (1981)



SCHRIFTEN ZUR QUANTITATIVEN BETRIEBSWIRTSCHAFTSLEHRE

Technische Hochschule Darmstadt

1995

- 1/95 **Stadtler, Hartmut**: Hierarchische Produktionsplanung. Mai 1995, *erschienen in: Kern, W.; H.-H. Schröder und J. Weber (Hrsg.): Handwörterbuch der Produktionswirtschaft, 2. Aufl., Schäffer-Poeschel, Stuttgart, Sp. 631 - 652.*
- 2/95 **Undt, Hans-Jürgen und Armin Scholl**: Erzeugung von Testdaten für Reihenfolgeprobleme bei Variantenfließfertigung. Juni 1995.
- 3/95 **Sondergeld, Lutz und Stefan Voß**: A Star-shaped Diversification Approach in Tabu Search. Januar 1995, *erschienen in: Osman, I.H. und J.P. Kelly (Hrsg.): Meta-Heuristics: Theory & Applications, Kluwer, Boston, 1996, S. 489 - 502..*
- 4/95 **Scholl, Armin; Robert Klein und Wolfgang Domschke**: Pattern Based Vocabulary Building for Effectively Sequencing Mixed Model Assembly Lines. August 1995.
- 5/95 **Amberg, Anita; Wolfgang Domschke und Stefan Voß**: Capacitated Minimum Spanning Trees: Algorithms Using Intelligent Search. August 1995, *erschienen in: Combinatorial Optimization: Theory and Practice 1, 1996, S. 9 - 39.*
- 6/95 **Strahinger, Susanne**: Zum Begriff des Metamodells. September 1995, *erschienen als: WI-Depot-Beitrag 96/3, Wirtschaftsinformatik 38/5, 1996, S. 545.*
- 7/95 **Domschke, Wolfgang; Robert Klein und Armin Scholl**: Antizipative Leistungsabstimmung bei moderner Variantenfließfertigung. Oktober 1995, *erschienen in: Zeitschrift für Betriebswirtschaft 66 (1996), Heft 12, S. 1465 - 1491.*
- 8/95 **Strahinger, Susanne**: Eine kardinalitätsbezogene Erweiterung der Entity-Relationship-Modellierung. Oktober 1995.
- 9/95 **Stadtler, Hartmut**: Reformulations of the Shortest Route Model for Dynamic Multi-Item Multi-Level Capacitated Lotsizing. September 1995.

1996

- 1/96 **Domschke, Wolfgang und Gabriela Krispin**: Location and Layout Planning - A Survey. Januar 1996, *erscheint in: OR Spektrum.*
- 2/96 **Strahinger, Susanne**: Konzept metamodellbasierter Methodenvergleiche. Februar 1996.
- 3/96 **Scholl, Armin; Robert Klein und Christian Jürgens**: BISON: A Fast Hybrid Procedure for Exactly Solving the One-Dimensional Bin Packing Problem. März 1996, *erschienen in: Computers & Operations Research 24, S. 627 - 645.*

- 4/96 Domschke, Wolfgang; Robert Klein und Armin Scholl: Tabu Search - Eine intelligente Lösungsstrategie für komplexe Optimierungsprobleme. April 1996, *erschienen in: WiSt 25 (1996), Heft 12, S. 606 - 610.*
- 5/96 Stadler, Hartmut: On the Equivalence of LP Bounds Provided by the Shortest Route and the Simple Plant Location Model Formulation for Dynamic Multi-Item Multi-Level Capacitated Lot-Sizing. Mai 1996.
- 6/96 Domschke, Wolfgang; Robert Klein und Armin Scholl: Tabu Search - Durch Verbote zum schnellen Erfolg. August 1996, *erschienen als: Taktische Tabus - Tabu Search: Durch Verbote schneller optimieren. c't - Magazin für Computertechnik, Heft 12 (1996), S. 326 - 332.*
- 7/96 Stadler, Hartmut: Optimal Dimensions for Automated Storage/Retrieval Systems. Juli 1996, *erscheint in: Proceedings of 1996 International Material Handling Research Colloquium.*

1997

- 1/97 Scholl, Armin; Gabriela Krispin; Robert Klein und Wolfgang Domschke: Branch and Bound - Optimieren auf Bäumen: je beschränkter, desto besser. Januar 1997, *erscheint in: c't - Magazin für Computertechnik.*
- 2/97 Scholl, Armin; Wolfgang Domschke und Robert Klein: Logistik: Aufgaben der Tourenplanung. Februar 1997, *erscheint in: WISU.*
- 3/97 Scholl, Armin; Wolfgang Domschke und Robert Klein: Logistik: Methoden der Tourenplanung. Februar 1997, *erscheint in: WISU..*
- 4/97 Klein, Robert und Armin Scholl: Computing Lower Bounds by Destructive Improvement - an Application to Resource-Constrained Project Scheduling. März 1997.
- 5/97 Scholl, Armin und Robert Klein: Balancing Assembly Lines Effectively - a Computational Comparison. April 1997.
- 6/97 Scholl, Armin und Robert Klein: Improving the Efficiency of JIT Assembly Lines: the U-Line Balancing Problem. Mai 1997.
- 7/97 Scholl, Armin und Robert Klein: Investitionsprogrammplanung bei Obelix GmbH & Co. KG. Juli 1997.

